If n be an odd No. $\beta_{\frac{1}{2}(n+1)}$ will be the middle term and its mean value will be $\frac{1}{2}a$. If n be an even No. $\beta_{\frac{1}{2}n}$ and $\beta_{\frac{1}{2}(n+1)}$ are middle terms; their mean values are $\frac{1}{2}a+\frac{\frac{1}{2}a}{n+1}$ and $\frac{1}{2}a-\frac{\frac{1}{2}a}{n+1}$, and the mean of these values is $\frac{1}{2}a$. The sum of n values is $\frac{1}{2}an$, and hence the mean of all the values is $\frac{1}{2}a$.

Suppose now we take the values of the three tickets, as estimated per question by the independent voters, and that these values are as the Nos. 1, 2, 3. Then, substituting n by 3, a by 100, and r severally by 1, 2, 3, we shall have for the mean value of the most worthy ticket, $\frac{3}{4}\times 100 = 75$,

for the mean value of the next best, $\frac{1}{2}\times 100 = 50$,

for the mean value of the third " , $\frac{1}{4}\times 100 = 25$;

while the *mean* of all the values will be $\frac{1}{3}\times\frac{3}{2}\times 100 = 50$.

SOLUTION OF A PROBLEM.

BY WILLIAM HOOVER, SUPERINTENDENT OF SCHOOLS, WAPAKONETA, O.

Problem.—A triangle is formed by joining the centers of the escribed circles of a triangle; the same is done for the new triangle, and so on *ad infinitum.* Show that the final triangle is equilateral.

Solution.—If A be one of the angles of the given triangle, and A_1, A_2, A_3, &c., the successive corresponding angles, we have

$A_1 = (\frac{1}{2})(\pi-A)$, $A_2 = (\frac{1}{2})^2(\pi+A)$, $A_3 = (\frac{1}{2})^3(3\pi-A)$, $A_4 = (\frac{1}{2})^4(5\pi+A)$.

The fraction for the nth angle is $(\frac{1}{2})^n$, the coefficient of π is the general term of the recurring series 1, 1, 3, 5, &c., $\frac{1}{3}[2^n + (-1)^{n-1}]$, and of A, it is $(-1)^n$. Hence

$$A_n = \frac{1}{2^n}\left[\tfrac{1}{3}\pi[2^n+(-1)^{n-1}]+(-1)^nA\right].$$

When n is infinite $A_n = \frac{1}{3}\pi$, and the triangle is equilateral.

SOLUTION OF PROB. 282 *WITH CONDITION* (3) *CHANGED.*

BY CHAS. H. KUMMELL, U. S. LAKE SURVEY.

The solution of this problem (see pp. 25, 26) is in accordance with the given conditions. However it cannot be admitted, as assumed in condition (3), that it is equally probable that only one loom should be out of order as

that any other number should be out of order. I propose to solve the problem by assuming a looms, each netting m dollars a day when in order, x weavers, each getting n dollars a day, whether working or not, and p the probability of a single loom being out of order. We have then to take the general mean of the following cases:

No. of looms in order.	Profits.	Probability.	
a	mx	$(1-p)^a$	$= P_a$
$a-1$	mx	$\frac{a}{1}\left(1-p\right)^{a-1} p$	$= P_{a-1}$
$a-2$	mx	$\frac{a}{1}\cdot\frac{a-1}{2}\left(1-p\right)^{a-2} p^2$	$= P_{a-2}$
.	.	. . . .	.
x	mx	$\frac{a}{1}\cdot\frac{a-1}{2}\cdots\frac{x+1}{a-x}\left(1-p\right)^{x} p^{a-x}$	$= P_x$
$x-1$	$mx-(m+n)$	$\frac{a}{1}\cdot\frac{a-1}{2}\cdots\frac{x}{a-x+1}\left(1-p\right)^{x-1} p^{a-x+1}$	$= P_{x-1}$
$x-2$	$mx-2(m+n)$	$\frac{a}{1}\cdot\frac{a-1}{2}\cdots\frac{x-1}{a-x+2}\left(1-p\right)^{x-2} p^{a-x+2}$	$= P_{x-2}$
.	.	. . . .	.
1	$mx-(x-1)(m+n)$	$\frac{a}{1}\left(1-p\right)p^{a-1}$	$= P_1$
0	$mx-x(m+n)$	p^a	$= P_0$

We have then the average profit

$$y_x = mx-(m+n)\Sigma_1^x[sP_{x-s}].$$

If this is a maximum then

$$y_x - y_{x-1} = \Delta y_x = m-(m+n)\Sigma_1^x [P_{x-s}] \text{ is small and positive,}$$

$$\& \; y_{x+1} - y_x = \Delta y_{x+1} = m-(m+n)\Sigma_1^{x+1} [P_{x+1-s}] \text{ is small and negative,}$$

Placing $a = 60$, $m = 3$, $n = 1$ and $p = 0.1$ we find $\Delta y_{56} = +\,0.021$ and $\Delta y_{57} = -0.113$. Therefore $x = 56$ weavers.

ANSWERS TO QUERIES IN NUMBER FOUR.

"QUERY BY PROF. A. HALL.—(1). If v be the potential function we have the equation given by Laplace

$$\frac{d^2 v}{dx^2} + \frac{d^2 v}{dy^2} + \frac{d^2 v}{dz^2} = 0,$$

which holds for a point outside the attracting body. For a point inside this body we have the equation given by Poisson,